Some issues on 1d-TEM inversion utilizing various multiple data strategies

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Summary

For many years since the early 1980's, inloop TEM inversions were not only performed but recommended by manufacturers of such equipment including the use of smooth over-parametrized models.. For various reasons, we have opposed this simplistic approach and have sought more precise techniques to provide higher resolution models with appropriate physical constraints. In previous research, we have studied the importance of correct system representation and how to provide more precise under-parametrized and geologically constrained models. Now we have begun exploring the importance of utilizing multiple data elements to provide accurate results.

Introduction

Time-Domain Electromagnetic (TEM) sounding techniques are successfully applied to various areas of geoexploration. TEM systems induce electical currents in the earth using electromagnetic induction. A time varying magnetic field is created using a coil or loop of wire. Faraday's law of induction tells us that a changing magnetic field will produce an electric current, which in turn will create an magnetic field. Therefore, the primary magnetic field from the transmitter loop will generate a secondary electric current in the earth. TEM techniques utilize the information about the secondary magnetic field produced by those secondary electric currents in the earth. The magnitude and rate of decay of those currents depend on the conductivity of the medium and on the geometry of the conductive layers. The induced currents in the earth are initially concentrated immediately below the transmitter loop. With time, those currents will diffuse down and away from the transmitter. The behaviour of the currents in the ground is often described as smoke rings in Nabighian (1979). In resistive media the currents will diffuse rapidly whilst the currents will diffuse slowly in conductive media. It is evident that making measurment of the output voltage of the receiver coil or the output of a magnetometer at sucessive times will reveal the electrical resistivity of the earth at sucessively greater depths.

A number of authors have made efforts to generate enhanced resolutions of inverted models by utilizing various components of data. In Zhang (2001) the merits of joint inversion of surface and borehole data were studied. They have noted that the surface data have a higher signal-to-noise ratio at early times while the borehole data have a higher signal-to-noise ratio at late times. Therefore, joint interpretation of surface and borehole data may give more information about the geological targets of interest. In Auken (2004) a 1-D laterally constrained inversion utilizing information on the neighboring 1D-models was implemented on TEM data to yield an enhanced resolution of subsurface. Viezzoli (2008) generated a 3d-model of EM data utilizing a 1d-inversion. In Groom 2005 we applied inversion on in-loop and out-loop data individually and concluded that out-loop data may resolve deeper structures better than in-loop data.

In this paper, we formulate the 1d-inversion as a weighted non-linear least-squares minimization problem. The weighted non-linear least squares are suitable for dealing with the cases where the measurements have different uncertainties. Our inversion incorporates the data of multiple components and multiple stations. We have developed a strategy for selecting a weighting matrix so that each observed data is well represented in the final parameter estimates. We demonstrate with synthetic examples that incorporating the data of multiple components and multiple stations into the inversion help resolve the ambiguity due to the non-uniqueness of the models that fit the data and thus leads to more meaningful geological models.

Forward Modelling

We have developed algorithms to simulate the EM response of a layered earth model incorpating important system parameters. The EM responses were computed for systems with various current waveforms and survey configurations with appropriate frequency bandwidths. We allow both in-loop and out-of-loop mesaurements and provide for both moving and fixed transmitter configurations with arbitrary location and orientation of receivers. The method also allows for systems having dipole transmitters (small coil) as well as measured magnetic fields via direct magnetometer measuements or processed magnetic fields. This technique encorporates the periodic nature of actual systems as well as the actual bandwidth of commercial systems. To compute the time domain response, the frequency domain response is generated first. Calculation of the frequency domain response involves taking into consideration of the characteristics of the earth response, as well as the transmitter and receiver geometries. One of the major tasks in the simulation is to compute an accurate earth response in frequency domain. The time domain response is obtained via Fourier series with the assumption that current waveforms are periodic which is a close representation particularly for ground data. The number of the harmonics utilized in the transformation determines the bandwidth of the time domain data. We can simulate the responses excited with a wide variety of current waveforms such as adjusted square waves used by Geonics, Crone, TerraTeM and Zonge etc, half-sine waveforms utilized by GeoTEM and MegaTEM, triangle-pulse waveform used by AeroTem, sawtooths waveform sused by UTEM, as well as a sine-on sine-off pulse as used by VTEM.

Inversion

We adopt the notation consistent with fitting a model to *n* pieces of data $\bar{d} = (d_1, d_2, \cdots, d_n)$ utilizing model parameters $x = (x_1, x_2, \cdots, x_p)$. In our particular case, these parameters consist of the resistivity and thickness of a layered earth. Let $\hat{d} = (\hat{d}_1(x), \hat{d}_2(x), \cdots, \hat{d}_n(x))$ be the predicted values from an estimated model. We utilized our forward modeling techniques mentioned above to compute these predicted values. The residual is defined as $R(x) = (r_1(x), r_2(x), \cdots, r_n(x))$ with $r_i(x) = d_i - \hat{d}_i(x)$. We wish to solve a non-linear weighted least squares problem:

$$\text{Minimize } f(x) = \frac{1}{2}\|WR\|^2 = \frac{1}{2}R^T W^T WR,$$

where *W* is a weight matrix and is diagonal.

We utilize a quasi-Newton technique to solve the minimization problem. The Newton model is based on the assumption that $f(x)$ can be adequately modeled by a quadratic. At a point during the inversion process, we construct a quadratic approximation to the objective function $f(x)$ that matches the first and the second derivative values at that point. However, this process requires a good approximation to $\nabla^2 f(x)$, the Hessian of $f(x)$. Suppose that $J(x)$ is the Jacobian of $R(x)$. In our case, the Jacobian $J(x)$ is computed numerically with a forward difference method. Then the Hessian of $f(x)$ can be expressed as

$$\nabla^2 f(x) = J^T(x) W^T WJ(x) + \sum_{i=1}^{n} w_i^2 r_i(x) \nabla^2 r_i(x).$$

At iteration *k*, the first term in this expression can be obtained readily from the Jacobian $J(x_k)$. As a matter of fact, it is often possible to ignore the second-order term $\sum_{i=1}^{n} w_i^2 r_i(x_k) \nabla^2 r_i(x_k)$ of the Hessian and $J^T(x) W^T WJ(x)$ is a sufficiently good Hessian approximation Dennis (1977). In such a case, the commonly called Gauss-Newton method is applicable. The new point x_{k+1} can be found by solving the linear least squares problem:

$$\min_{x_{k+1}} \left(J(x_k)(x_{k+1} - x_k) + R(x_k)\right)^T W^T W \left(J(x_k)(x_{k+1} - x_k) + R(x_k)\right)$$

This minimization problem can be solved with the normal equation system

$$J^T(x_k) W^T WJ(x_k)(x_{k+1} - x_k) = J^T(x_k) W^T WR(x_k).$$

In large-residual problems, the quadratic model is an inadequate representation of the function $f(x)$ as the second-order part of the Hessian is too significant to ignore. We utilized a technique developed in Dennis (1981) to approximate the second term $\sum_{i=1}^{n} w_i^2 r_i(x_k) \nabla^2 r_i(x_k)$ with a symmetric matrix S_k and the overall Hessian approximation is $B_k = J^T(x_k) W^T WJ(x_k) + S_k$. Then a new point x_{k+1} is obtained utilizing a trust-region approach. Updates to S_k are devised so as to mimic the behavior of $\sum_{i=1}^{n} W_i^2 r_i(x_k) \nabla^2 r_i(x_k)$. This requires that S_{k+1} be symmetric and the difference $S_{k+1} - S_k$ be minimized in terms of a weighted Frobenius norm. The update formula is:

$$S_{k+1} = S_k + \frac{(y_k - S_k \Delta x_k) u^T + u (y_k - S_k \Delta x_k)^T}{\Delta x_k^T u} - \frac{\Delta x_k^T (y_k - S_k \Delta x_k) u u^T}{\left(\Delta x_k^T u\right)^2},$$

where $y_k = J^T(x_{k+1}) W^T R(x_{k+1}) - J^T(x_k) W^T WR(x_{k+1})$,

$\Delta x_k = x_{k+1} - x_k$, $u = J^T(x_{k+1}) W^T WR(x_{k+1}) - J^T(x_k) W^T WR(x_k)$.

In case of a zero residual target, the matrix S_k thus constructed does not necessarily become small. To avoid such a difficulty, replace S_k by $\tau_k S_k$ prior to the computation with $\tau_k = \min\{1, |\Delta x_k^T y_k| / |\Delta x_k^T S_k \Delta x_k|\}$. Moreover, the matrix S_k is omitted from the Hessian approximation in the case that the Gauss-Newton model produces a sufficiently good new point:

$$x_{k+1} = x_k + \left(J^T(x_k) W^T WJ(x_{k+1})\right)^{-1} J^T(x_k) W^T WR(x_k).$$

With a layered earth model, the EM responses of the late offtime channels are usually smaller than the responses of the early time channels. Mover, if the data consists of multiple components or multiple stations, there will be a significant variation of the data amplitude. The weighted matrix *W* determines to what extent each observation in the data set influences the final parameter estimates. Optimizing the weighted fitting criterion to find the parameter estimates allows the weights to determine the contribution of each observation to the final parameter estimates. In real applications, weights are not known exactly and therefore estimated weights must be used instead. Application of a good weight matrix may improve the condition number of the first matrix $J^T(x) W^T WJ(x)$ of the Hessian and the approximate Hessian B_k discussed above and therefore helps enhance the feasibility that $f(x)$ can be modeled with a quadratic. Usually, weighted least squares utilize weights that are inversely proportional to the standard deviation of an observed data. The *i*-th diagonal element w_i of W is set to be $1/\sigma_i$, where σ_i is the standard deviation of the *i*-th data. We assume that the deviation of the data is proportional to a modified data value, that is, $\sigma_i = \lambda |d_i|^q$, $i = 1,2 \cdots n$, for some q, $0 \le q < 1$. Note that λ is independent of *i* and therefore it can be omitted since the weight for each observed data is given relative to the weights of the other observed data. Consequently, this will lead to a weight matrix with $w_i = 1/|d_i|^q$. Note that the larger the value for *q* is, the more weight will be imposed on low amplitude data. We tested many choices for *q*, which has led us to empirically set

$$q = \begin{cases} 1 - \dfrac{\ln 20}{\ln |d|^{\max} - \ln |d|^{\min}}, & |d|^{\max} > 20 |d|^{\min} \\ 0, \quad |d|^{\max} <= 20 |d|^{\min} \end{cases}$$

where $|d|^{\max} = \max\{|d_i|, 1 \le i \le n\}$, $|d|^{\min} = \min\{|d_i| \ne 0, 1 \le i \le n\}$, that is, $|d|^{\max}$ and $|d|^{\min}$ are the maximum and the minimum amplitude of the non-zero observed data. In case of $|d|^{\max} \le 20 |d|^{\min}$, no weight is imposed and the minimization problem becomes a unweighted least-squares problem.

Examples

The multi-station inversion technique was applied to a synthetic ground TEM dataset. For the synthetic survey, a 400 x 400 m loop centered at (0, 0) was used. Receiver locations were every 100 m on a north-south line from 0N to 800N (except at 200N where the north side of the loop is located). The base frequency of the system was 30 Hz, and there were 20 off-time channels.

Table 1: True Model for Synthetic Data

Resistivity (Ωm)	Thickness (m)	Depth to Bottom (m)
500	400	-400
50	50	-450
500		

Synthetic data for this survey was created for the following 1D model: a 500 Ωm half-space with a 50 m thick, 50 Ωm conductor at 400 m depth (Table 1). Quasi-Newton inversions were performed on this data. The starting model was a 500 Ωm half-space with a 50 Ωm, 50 m thick conductor at -160 m was used. That is, the position of the conductor has been shifted up 240m from the true model. Inversions were unconstrained. With the conductor of the starting model at 160 m depth, a single-station inversion finds a good result only at some locations, though if the starting conductor is pushed deeper to 200 m, all of the stations will find a good result.

The initial inversion (Figure 1) was a single-station inversion on the vertical component with the starting model outlined above. A good model was obtained at only two data points: 300N and 600N. The inversion results at these two points were close to the true model: the resistivities of the first and third layers were close to 500 Ωm, and the conductance of the second layer was close to 1 S, as in the model. The data is not very sensitive to the conductivity of this layer, but is sensitive to the conductance. At five of the other stations (0N, 100N, 500N, 700N, and 800N), the inversion results are similar to each other, but the models do not fit the data as well. In these models, the resistivity of the top layer is close to 500 Ωm, but the second layer is a strong resistor at 200 m, followed by a somewhat conductive layer of about 200 Ωm. These models fit the data well at early times, but do not match its curvature at mid-late times (Figure 2).

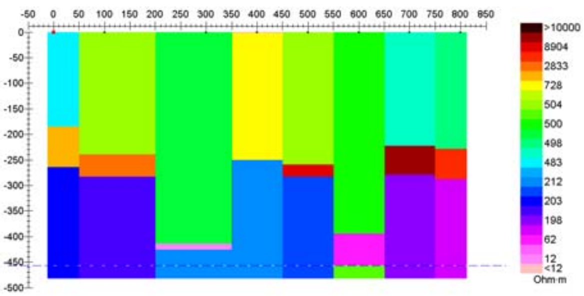

Figure 1: Stacked 1D inversions results for the single station inversion of Hz.

A further single station inversion was performed on the vertical component data, but the result of the previous point was used as the starting model, rather than using the same starting model at each point. This inversion had worse results: at the two points that had good results in the initial inversion, the results were poor. At every point, a model with a very resistive second layer at about 200 m depth was found.

Results for a two-component inversion (Hx-inline and Hz-vertical) were improved over the inversions on Hz only. Good results were obtained at five of the eight stations, but results were no better at 0N (where Hx has no response from the layered earth), and 700N and 800N.

The use of the multi-station inversion technique significantly improved inversion results for Hz. In a multi-station inversion using all eight stations, excellent results were obtained (Figure 2). The model fits the data well and is close to the true model (Table 2). The misfit of the model to the data was below 1% in only five iterations. Another multi-station inversion, utilizing only stations 0N and 800N had good results as well. Both of these locations had poor results in the single-station inversion, but when used together in the multi-station inversion, a much better model was found.

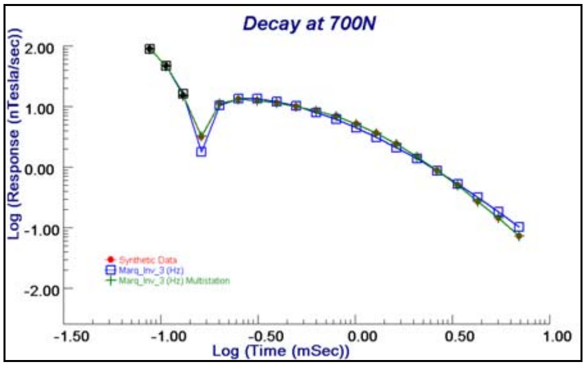

Figure 2: Decay at 700N. Red is the synthetic data. Blue is the result of the single-station inversion on Hz at 700N. Green is the result of the multi-station inversion on all eight stations.

Table 2: Model from Multi-station Inversion

Resistivity (Ωm)	Thickness (m)	Depth to Bottom (m)
500	401	-401
49	49	-450
496		

This result is due to the ambiguity in a model of a single station data whereas the multi-station data seems to have only one reasonable model.

Conclusions

It is possible to apply multiple TEM data elements to an under-parametrized inversion for a single multi-layer, model. Such multiple TEM data elements can be data from multiple stations and/or data from different data components. However, it is our experience that the use of suitable weighting terms is critical to the process.

The extension to multiple data components has a variety of benefits including the ability to include more parameters in the under-parametrized model, better signal to noise characteristics in the inversion process and large reductions in the number of suitable models.